SECTION - I

(1) $252 = 2^2 \times 3^2 \times 7$

$308 = 2^2 \times 7 \times 11$

$198 = 3^2 \times 2 \times 11$

LCM $= 2^2 \times 3^2 \times 7 \times 11 = 2772$

∴ A, B & C will meet after 2772 secs.

(2) $(K-1)(-3)^2 + K(-3) + 1 = 0$

$\Rightarrow 9K - 9 - 3K + 1 = 0$

$\Rightarrow 6K - 8 = 0$

$\Rightarrow K = \frac{\cancel{8}\,4}{\cancel{6}\,3} = \frac{4}{3}$

OR

$S = \frac{100}{2}(100 + 1)$

$\Rightarrow 50(101) \Rightarrow 5050$

∴ The sum of of the first 100 natural no. is 5050.

3. $4^{x+y} = 256$ & $256^{x-y} = 4$

$\Rightarrow 4^{x+y} = (4)^4$ $\qquad (4)^{4(x-y)} = (4)^1$

$\Rightarrow x + y = 4$ —— (i) $\qquad \Rightarrow 4x - 4y = 1$ —— (ii)

multiplying 4 in eqn (i)

$4x + 4y = 16$ —— iii

adding (iii) & (ii)

$4x - 4y = 1$

$4x + 4y = 16$

$8x = 17$

$\Rightarrow x = \frac{17}{8}$

$y = 4 - \frac{17}{8} \Rightarrow \frac{32 - 17}{8}$

$y = \frac{15}{8}$

∴ They will intersect at the point $\left(\frac{17}{8}, \frac{15}{8}\right)$

(4) In ΔDEC, & ΔABC,

$\angle C$ (common) & $\angle D = \angle B$ (corresponding $\angle s$)

$\Delta DEC \sim \Delta ABC$

$\Rightarrow \frac{AC}{DC} = \frac{BC}{EC}$

$\Rightarrow \frac{AD + CD}{CD} = \frac{BE + EC}{EC}$

$\Rightarrow 1 + \frac{AD}{CD} = 1 + \frac{BE}{EC}$ (Hence proved)

OR

PQ ∥ NO, By BPT;

$\frac{MP}{PN} = \frac{MQ}{OQ} \Rightarrow \frac{4}{13} = \frac{MQ}{15.6} \Rightarrow MQ = \frac{4 \times 15.6}{13} \Rightarrow MQ = 4.8$ cm

5. $\tan\theta = \frac{5}{3}$ NOW, $\frac{\cancel{5}3\sin\theta - 5\cos\theta}{3\sin\theta + 5\cos\theta}$

$\Rightarrow \frac{3\frac{\sin\theta}{\cos\theta} - \frac{5\cos\theta}{\cos\theta}}{3\frac{\sin\theta}{\cos\theta} - \frac{5\cos\theta}{\cos\theta}} \Rightarrow \frac{3\tan\theta - 5}{3\tan\theta + 5} \Rightarrow \frac{3 \times \frac{5}{3} - 5}{3 \times \frac{5}{3} + 5} \Rightarrow \frac{5-5}{5+5} \Rightarrow \frac{0}{10} \Rightarrow 0$

6. AOBP is a cyclic quadrilateral,

$\angle AOB + \angle APB = 180°$,

$110° + \angle APB = 180°$

$\angle APB = 70°$

7. $\theta = 60°$ $r = 15$ cm

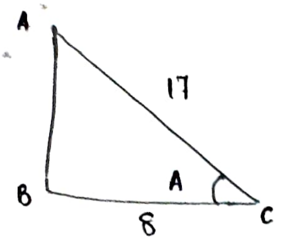
$\frac{\cancel{60}}{\cancel{360}_6} \times \frac{22}{7} \times 15 \times 15 = 117.75$ cm²

∴ The minute hand will cover an area of 117.75 cm².

8. Total outcome = 36

Favourable outcome = 12

Probability = $\frac{\cancel{12}}{\cancel{36}_3} = \frac{1}{3}$

9. 

$AB^2 = 289 - 64$
$= 225$

$AB = 15$

$\sin A = \frac{15}{17}$, $\cos A = \frac{8}{17}$.

$$\frac{3 - 4\left(\frac{15}{17}\right)^2}{4 \times \left(\frac{8}{17}\right)^2 - 3} \Rightarrow \frac{3 - 4 \times \frac{225}{289}}{4 \times \frac{64}{289} - 3} = \frac{867 - 900}{256 - 867} = \frac{-33}{-611} = \frac{33}{611}$$

OR

$\tan 30° + \sin 60° . \sec 60°$

$$\Rightarrow \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{\cancel{2}} \times \cancel{2} \Rightarrow \frac{1+3}{\sqrt{3}} \Rightarrow \frac{4}{\sqrt{3}}.$$

10. ROSP is a cyclic quadreilaterral,

$\angle ROS + \angle RPS = 180°$,

$\angle ROS + 25° = 180°$

$\angle ROS = 155°$.

11. $(a_1)^3 + (a_2)^3 + (a_3)^3 = (A)^3$

$\Rightarrow 4^3 + 5^3 + 6^3 = A^3$

$\Rightarrow 64 + 125 + 216 = A^3$

$\Rightarrow \sqrt[3]{405} = A$

$\Rightarrow A = 7.4$ (approx)

12. The distance betn 2 parrallel tangents = Diameterr of circle

∴ The distance betn the 2 parrallel tangents is $2 \times 5 = 10$ cm.

13. $(-4)^2 - 4(5)(3)$

$\Rightarrow 16 - 60$

$\Rightarrow -44$

$D < 0$

There are no real roots exist.

14. Total outcome = 36

Favorable outcome = 4

Probability $\Rightarrow \frac{4}{36} \Rightarrow \frac{1}{9}$

15. $2\pi r \Rightarrow 2 \times \frac{22}{7} \times \frac{28}{1} \Rightarrow 176$

$\Rightarrow 4 \times \text{side} = 176$

$\Rightarrow \text{Side} = 44$ cm

16. LHS $= 3x^2 + 14x - 5$

$3(-5)^2 + 14(-5) - 5$

$\Rightarrow 75 - 70 - 5$

$\Rightarrow 75 - 75 \Rightarrow 0$

$\therefore$ -5 is a solution of quadratic eqn $3x^2 + 14x - 5 = 0$.

OR

#. $D = 0$

$(-K)^2 - 4(3)(K) = 0$

$K^2 - 12K = 0$

$K(K - 12) = 0$

$K = 0$ or $K = 12$

$\therefore$ The required value of K is 12.

SECTION - II

17 (a) (I) cylindrical
(b) (II) 282.6 m^3
(c) (III) 28 . 26 m^2
(d) (I) 216. 66 m^2
(e) (II) 21413.0

20 (a) (IV) 3m
(c) (I) 2.16 m^2
(d) (III) Trapezium
(e) (III) do not say anything.

18 (a) (II) [0, 1]
(b) (I) 0
(c) (III) 1
(d) (II) $\frac{11}{42}$

(e) (I) $\frac{31}{42}$

21. $12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

LCM $= 2^2 \times 3 \times 5 \times 7 = 420$

HCF $= \cancel{2}\ 3$

OR

$$\frac{47}{2^5 \times 5^3} = \frac{47}{2^2(2^3 \times 5^3)} = \frac{47}{4(1000)} = \frac{11.75}{1000} = 0.01175$$

It will terminate after 5 places.

22) $a+d = 2$ — (i)

$a+6d = 22$ — (ii)

Subtracting (i) from (ii)

$a+6d = 22$

$a+d = 2$

(−) (−) (−)

$5d = 20$

$\Rightarrow d = 4$

$\Rightarrow a = -2$

$\Rightarrow -2 + 34(4)$

$\Rightarrow 136 - 2$

$\Rightarrow 134$

$S_{35} = \frac{35}{2}(-2+134) \Rightarrow \frac{35}{\cancel{2}} \times \cancel{132}^{66} \Rightarrow 2310$

23)

Marks obtained	No. of students	x_i	$f_i x_i$
0-10	14	5	70
10-20	8	15	120
20-30	15	25	375
30-40	21	35	735
40-50	9	45	405
50-60	8	55	440
	$\Sigma f_i = 75$		$\Sigma f_i x_i = 2145$

Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2145}{75} = 28.6$

(24) LHS $= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$$\Rightarrow \frac{\sin\theta(1-2\sin^2\theta)}{2\cos\theta(2\cos^2\theta-1)} \Rightarrow \frac{\sin\theta(\cos^2\theta+\sin^2\theta-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-\sin^2\theta-\cos^2\theta)}$$

$$\Rightarrow \frac{\sin\theta(\cancel{\cos^2\theta-\sin^2\theta})}{\cos\theta(\cancel{\cos^2\theta-\sin^2\theta})} = \frac{\sin\theta}{\cos\theta} = \tan\theta = RHS$$

(Hence proved)

OR

LHS: $\cot A + \tan A$

$$\Rightarrow \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A} \Rightarrow \frac{\cos^2 A + \sin^2 A}{\sin A \cos A} = \frac{1}{\sin A\cos A}$$

RHS: $\sec A \operatorname{cosec} A$

$$\Rightarrow \frac{1}{\cos A} \times \frac{1}{\sin A} \Rightarrow \frac{1}{\sin A \cos A}$$

$\therefore$ LHS = RHS

(Hence proved)

25. $f(x) = px^2 + qx + r \Rightarrow \alpha+\beta = -\frac{q}{p}$; $\alpha\beta = \frac{r}{p}$

$$\frac{1}{p\alpha+q} + \frac{1}{p\beta+q} \Rightarrow \frac{p\beta+q+p\alpha+q}{p^2\alpha\cdot\beta + p\alpha\cdot q + p\beta\cdot q + q^2}$$

$$\Rightarrow \frac{p(\alpha+\beta)+2q}{p^2(\alpha\beta)+pq(\alpha+\beta)+q^2} \Rightarrow \frac{\cancel{p}\times -\frac{q}{\cancel{p}} + 2q}{p\cancel{p^2}\times\frac{r}{\cancel{p}} + \cancel{p}q\times -\frac{q}{\cancel{p}} + q^2} \Rightarrow \frac{-q+2q}{pr + (-q)^2 + q^2}$$

$$\Rightarrow \frac{q}{pr - \cancel{q^2} + \cancel{q^2}} \Rightarrow \frac{q}{pr}$$

$\therefore$ The value of $\frac{1}{p\alpha+q} + \frac{1}{p\beta+q}$ is $\frac{q}{pr}$.

26. Let $A(12,8)$, $B(-2,6)$, $C(6,0)$

AB = $\sqrt{(12+2)^2+(8-6)^2} \Rightarrow \sqrt{14^2+2^2} \Rightarrow \sqrt{196+4} \Rightarrow \sqrt{200} \Rightarrow 10\sqrt{2}$

BC = $\sqrt{(6+2)^2+(0-6)^2} \Rightarrow \sqrt{64+36} \Rightarrow \sqrt{100} = 10$

AC = $\sqrt{(12-6)^2+(8-0)^2} \Rightarrow \sqrt{36+64} \Rightarrow \sqrt{100} = 10$

$\because$ BC = AC, ΔABC is an isoceles triangle.

By, pythagoras theorum.

LHS = $AC^2 + BC^2$

$\Rightarrow (10)^2 + (10)^2 \Rightarrow 100 + 100 \Rightarrow 200$

RHS = AB^2

$\Rightarrow (10\sqrt{2})^2 \Rightarrow 200$

$\Rightarrow$ LHS = RHS

$\therefore$ (12,8), (-2,6) and (6,0) are vertices of a isosceles right angled triangle.

27. Let $\sqrt{n}$ be a rational no. where n isn't a rational no.

$\sqrt{n} = \frac{p}{q}$ where p & q are coprimes.

Squaring both the sides,

$n = \frac{p^2}{q^2} \Rightarrow nq^2 = p^2$

p^2 is divisible by n, p is divisible n — (I)

$p = nx$

$p^2 = n^2x^2$

$\Rightarrow nq^2 = n^2x^2$

$\Rightarrow q^2 = nx^2$

q^2 is divisible by n, q is divisible by n — (II)

eqn (I) & (II) contradicts the situation p & q are co-primes.

So, $\sqrt{n}$ is irrational.

(Hence proved)

OR

Length of the room = 8m 25cm = 825 cm

Breadth of the room = 6m 75 cm = 675 cm

Height of the room = 4m 50 cm = 450 cm

$825 = 3 \times 5 \times 5 \times 11$

$675 = 3 \times 3 \times 3 \times 5 \times 5$

$450 = 2 \times 3 \times 3 \times 5 \times 5$

HCF $= 3 \times 5 \times 5$

$= 75$

Hence, the length of the longest rod is 75 cm which can measure the 3 dimensions.

28. Let the cost of chair be x & table be y.
ATQ,

$$3x + 2y = 1850 \quad \text{---} (I)$$
$$5x + 3y = 2850 \quad \text{---} (II)$$

multiplying 5 in eqn (I) & 3 in eqn (II);

$\Rightarrow 15x + 10y = 9250 \quad \text{---} (III)$

$\Rightarrow 15x + 9y = 8550 \quad \text{---} (IV)$

Subtracting eqn (IV) from (III)

$$\begin{array}{r} \cancel{15x} + 10y = 9250 \\ \cancel{15x} + 9y = 8550 \\ \hline y = 700 \end{array}$$

$y = 700$ in eqn (I)

$3x + 2(700) = 1850$

$3x = 450$

$x = 150$

NOW, $7(150) + 3(700) = 3150$

$\therefore$ The cost of 7 chairs & 3 tables is ₹ 3150.

OR

$\frac{x+4y}{xy} = \frac{27xy}{xy} \Rightarrow \frac{1}{y} + \frac{4}{x} \Rightarrow 27 \quad \text{---} (I)$

$\frac{x+2y}{xy} = \frac{21xy}{xy} \Rightarrow \frac{1}{y} + \frac{2}{y} = 21 \quad \text{---} (II)$

Let $P = \frac{1}{y}$ & $q = \frac{1}{x}$

$\Rightarrow P + 4q = 27 \quad \text{---} III$

$\Rightarrow P + 2q = 21 \quad \text{---} IV$

Subtracting IV from III

$$\begin{array}{r} P + 4q = 27 \\ P + 2q = 21 \\ \hline 2q = 6 \end{array} \Rightarrow q = 3$$

$P + 4(3) = 27$

$P = 15$

$\frac{1}{y} = 15 \Rightarrow \boxed{y = \frac{1}{15}}$ & $\frac{1}{x} = 3 \Rightarrow \boxed{x = \frac{1}{3}}$

29: Side of square ABCD $= (7 \times 2 \times 3)$ cm $= 42$ cm
Area of square $= (42)^2 = 1764\ cm^2$
Area of one circle $= \frac{22}{7} \times 7 \times 7 = 154\ cm^2$

Area of the 9 circles $= 9 \times 154 = 1386\ cm^2$
$\therefore$ Area of the remaining portion is $1764 - 1386 = 378\ cm^2$

30.

Class interval	Frequency	cumulative frequency
129.5 - 139.5	4	4
139.5 - 149.5	9	13
149.5 - 159.5	18	31
159.5 - 169.5	28	59
169.5 - 179.5	24	83
179.5 - 189.5	10	93
189.5 - 199.5	7	100

$\frac{n}{2} = \frac{100}{2} = 50$

median class:- 159.5 - 169.5
$l = 159.5$, $cf = 31$, $h = 10$, $f = 28$
Median $= 159.5 + \left(\frac{50-31}{28}\right) 10$

$\Rightarrow 159.5 + \frac{190}{28} \Rightarrow 159.5 + 6.79 \Rightarrow 166.29$

$\therefore$ The median of the given data is 166.29.

31. (i) LHS $= PA \cdot PB$
$(PN - AN)(PN + BN) \Rightarrow (PN - AN)(PN + AN)$ (line segment from centre bisects the chord) ($\because AN = BN$)
$PN^2 - AN^2 =$ RHS
(Hence proved)

(ii) LHS $= PN^2 - AN^2$

$\Rightarrow (OP^2 - ON^2) - AN^2 \quad \Rightarrow OP^2 - (ON^2 + AN^2)$

$\Rightarrow \cancel{OR^2} \quad OP^2 - OA^2$

$\Rightarrow OP^2 - OT^2 \quad (OA = OT) \quad = RHS$

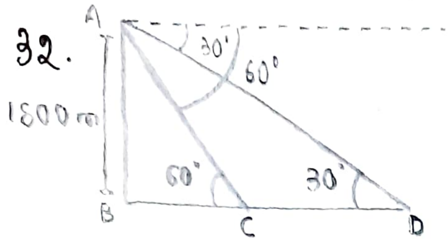
(Hence proved)

(iii) From (i) & (ii);

$PA \cdot PB = OP^2 - OT^2$

$= PT^2$ (RHS)

(Hence proved)

32. 

Let A be the position of the Aeroplane & C and D are the required position of the ships.

In right Δ ABC

$\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{1800}{BC} \Rightarrow BC = \frac{1800}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$BC = 600\sqrt{3}$ m

In right Δ ABD.

$\tan 30° = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{1800}{BD} \Rightarrow BD = 1800\sqrt{3}$ m

∴ The distance between the two ships are $1800\sqrt{3} - 600\sqrt{3} = 1200\sqrt{3}$ m.

33. $\frac{2x-3}{x-2} + \frac{2x-7}{x-4} = \frac{16}{3}$

$\Rightarrow \frac{2x-3(x-4)+(2x-7)(x-2)}{(x-2)(x-4)} = \frac{16}{3}$

$\Rightarrow \frac{2x^2-8x-3x+12+2x^2-4x-7x+14}{x^2-4x-2x+8} = \frac{16}{3}$

$\Rightarrow \frac{4x^2-22x+26}{x^2-6x+8} = \frac{16}{3}$

$\Rightarrow 12x^2-66x+78 = 16x^2-96x+128$

$\Rightarrow 12x^2-16x^2-66x+96x+78-128 = 0$

$\Rightarrow -4x^2+30x-50 = 0$

$\Rightarrow 4x^2-\cancel{+}30x+50 = 0$

$\Rightarrow 4x^2-20x-10x+50 = 0$

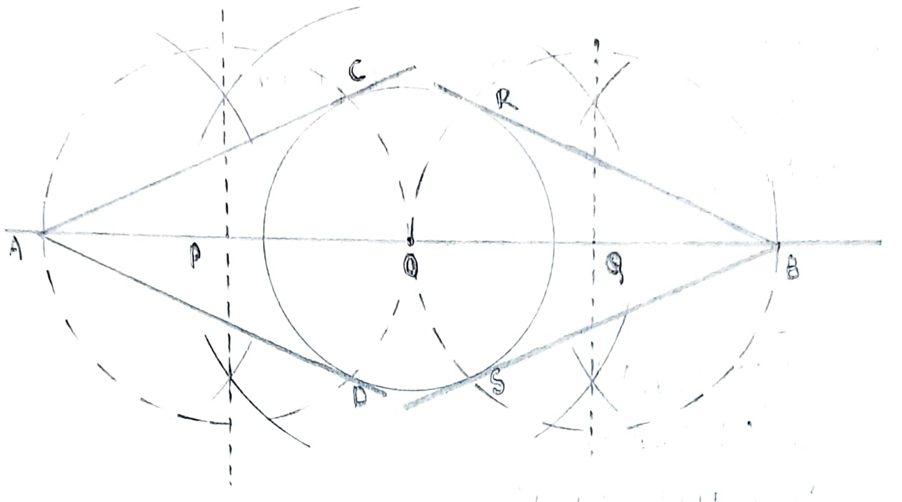
$\Rightarrow 4x(x-5)-10(x-5) = 0$

$(x-5)(4x-10)=0$

$x-5=0$ & $4x-10=0$

$\Rightarrow x=5$ and $x=\frac{10}{4}=\frac{5}{2}$

34.



Steps of construction:

* A circle of radius 2cm is drawn.
* line segment OA & OB of 5 cm was being drawn.
* $\perp$ of OA & OB was drawn which intersect OA at P & OB at Q.
* Taking P & Q as centre circles are drawn which intersect the circle of 2cm at D, C, R & S.
* AC, AD, BR & BS are joined. and hence are the required tangents.

35. LHS $= \cot^2 A\left(\frac{\sec A-1}{\sin A+1}\right)+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)$

$\Rightarrow \cot^2 A\left(\frac{\sec A-1}{1+\sin A}\times\frac{\sec A+1}{\sec A+1}\right)+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)$

$\Rightarrow \cot^2 A\left(\frac{\sec^2 A-1}{(1+\sin A)(1+\sec A)}\right)+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)$

$\Rightarrow \cot^2 A\left(\frac{\tan^2 A}{(1+\sin A)(1+\sec A)}\right)+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)$

$\Rightarrow \frac{1}{(1+\sin A)(1+\sec A)}+\sec^2 A\left(\frac{\sin A-1}{1+\sec A}\right)$

$\Rightarrow \frac{1+\sec^2 A(\sin A-1)(1+\sin A)}{(1+\sin A)(1+\sec A)}=\frac{1+\sec^2 A(\sin^2 A-1)}{(1+\sin A)(1+\sec A)}$

$\Rightarrow \dfrac{1 + \sec^2 A\,(-\cos^2 A)}{(1+\sin A)(1+\sec A)} \Rightarrow \dfrac{1-1}{(1+\sin A)(1+\sec A)} \Rightarrow \dfrac{0}{(1+\sin A)(1+\sec A)} \Rightarrow 0$

$\therefore$ LHS = RHS

(Hence proved)

OR

$\sqrt{\dfrac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}} + \sqrt{\dfrac{\operatorname{cosec} A + 1}{\operatorname{cosec} A - 1}} = 2\sec A$ (To prove)

~~Squaring both sides,~~
~~$\dfrac{\operatorname{cosec} A - 1}{\operatorname{cosec} A}$~~

RHS: $\sqrt{\dfrac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}} + \sqrt{\dfrac{\operatorname{cosec} A + 1}{\operatorname{cosec} A - 1}}$

Rationalising the denominator,

$\sqrt{\dfrac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} \times \dfrac{\operatorname{cosec} A - 1}{\operatorname{cosec} A - 1}} + \sqrt{\dfrac{\operatorname{cosec} A + 1}{\operatorname{cosec} A - 1} \times \dfrac{\operatorname{cosec} A + 1}{\operatorname{cosec} A + 1}}$

$\Rightarrow \sqrt{\dfrac{(\operatorname{cosec} A - 1)^2}{\operatorname{cosec}^2 A - 1}} + \sqrt{\dfrac{(\operatorname{cosec} A + 1)^2}{\operatorname{cosec}^2 A - 1}}$

$\Rightarrow \sqrt{\dfrac{(\operatorname{cosec} A - 1)^2}{\cot^2 A}} + \sqrt{\dfrac{(\operatorname{cosec} A + 1)^2}{\cot^2 A}} \Rightarrow \dfrac{\operatorname{cosec} A - 1}{\cot A} + \dfrac{\operatorname{cosec} A + 1}{\cot A}$

$\Rightarrow \dfrac{\operatorname{cosec} A - 1 + \operatorname{cosec} A + 1}{\cot A} \Rightarrow \dfrac{2 \operatorname{cosec} A}{\cot A} \Rightarrow \dfrac{2 \times \frac{1}{\sin A}}{\frac{\cos A}{\sin A}}$

$\Rightarrow 2 \cdot \dfrac{1}{\cos A} \Rightarrow 2\sec A = \text{RHS}$

(Hence proved)

36. Let the first term be a & common difference is d.

ATQ,

$a + 7d = 31$ ——— (1)

$\cancel{a} + 14d = \cancel{a} + 10d + 16$

$\Rightarrow 4d = 16$

$\Rightarrow d = 4$

putting $d = 4$ in eqn (1);

$a + 7(4) = 31$

$\Rightarrow a = 31 - 28 \Rightarrow a = 3$

(I) The AP Series is:

$3, 7, 11, 15, 19, 23 \ldots\ldots\ldots$

(II) 15^{th} term: $a + 14(d)$

$\Rightarrow 3 + 14(4) \Rightarrow 3 + 56 \Rightarrow 59$

$S_{15} = \frac{n}{2}(a + l)$

$\Rightarrow \frac{15}{2}(3 + 59) \Rightarrow \frac{15}{\cancel{2}} \times \overset{31}{\cancel{62}} \Rightarrow 465$

∴ The sum of the first 15 terms is 465.